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A study on Landau levels in thin films

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Abstract. In this paper, we study the energy levels of an electron moving in a thin film. This film is considered as a two-dimensional electron gas which is under the influence of a uniform external magnetic field B and a uniform external electric field E. Here, the magnetic field is perpendicular to the film. Also, in this paper, we have selected the Landau gauge, because this gauge is useful for working in rectangular geometries.

Keywords. thin film, landau gauge, energy levels, wavefunctions.

1 Introduction

A thin film is a liquid or solid matter such that one of its linear dimensions is very small in comparison with the other two dimensions. These films are produced by a process in the form of substrate growth. On the other words, they are made by means of sustaining an atomic or molecular flux to the surface of the substrate and subsequently by growing of the substrate. Substrate growth will either involve chemical reaction at the substrate such as discharge of ions, decomposition of a compound, reaction of a gas or liquid with substrate surface; or physical processes such as evaporation from a source and sputtering from a target, then condensation onto the substrate. Basically, we can classify thin films (arbitrarily) into: 1. Very thin films with thickness less than 50 A°, 2. Thin films with a thickness of between 50 A° to 5000 A°, 3. Thick films with a thickness of more than 5000 A°.

As mentioned above, thin films are planes with a thickness of 50 $A^{\circ}$to 5000 A°. Also, they are designed from a variety of materials including metals, insulators, and semiconduc-

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tors with a atomic precision. Thin films can be classified in the category of nanostructured coatings. The properties of the solid surface are modified for thin films, because the film structure and the limited thickness determine the physical properties. Thus, the effects of the geometric anisotropy and size should be studied [5,12].

During the 20th century, by improving deposition techniques in thin films, it has been enabled a wide range of technological breakthroughs in areas such as magnetic recording media, electronic semiconductor devices, LEDs, optical coatings (such as antireflective coatings), hard coatings on cutting tools, and for both energy generation (e.g. thin film solar cells) and storage (thin-film batteries). It is also being applied for pharmaceuticals, via thin-film drug delivery. In addition, thin films play an important role in the development and study of materials with new and unique properties. For example, multiferroic materials, and superlattices help us in study of the quantum confinement by creating two-dimensional electron states [1]- [14].

In this paper, we investigate the energy levels of an electron moving in a thin film. This film is supposed as a two-dimensional electron gas which is under the influence of a uniform external magnetic field B and a uniform external electric field E. Here, the magnetic field is perpendicular to the film. In this work, we have selected the Landau gauge, because this gauge is useful for working in rectangular geometries.

2 Landau Levels

In this section, we will review the quantum mechanics of free particles moving in a background magnetic field and the resulting phenomenon of Landau levels [15]. Throughout this discussion, we will neglect the spin of the electron. The reason is that in the presence of a magnetic field B, there is a Zeeman splitting between the energies of the up and down spins given by $\triangle = 2\mu_B B$ where $\mu_B = \frac{e\hbar}{2m}$ is the Bohr magneton. We will be interested in large magnetic fields where large energies are needed to flip the spin. This means that, if we restrict to low energies, the electrons act as if they are effectively spinless. Before we get to the quantum theory, we first need to briefly review some of the structure of classical mechanics in the presence of a magnetic field.

2.1 Canonical variables

The Lagrangian for a particle of charge $-e$ and mass m moving in a background magnetic field $B = \nabla \times A$ is

$$L = \frac{1}{2}m\dot{X}^2 - e\dot{X}.A, \tag{1}$$

under a gauge transformation $A \to A + \nabla\alpha$, the Lagrangian changes by a total derivative $L \to L - e\dot{\alpha}$. This is to ensure that the equations of motion remain unchanged under a gauge transformation. The canonical momentum arising from this Lagrangian is

$$P = \frac{\partial L}{\partial \dot{X}} = m\dot{X} - eA. \tag{2}$$

Now, we can compute the Hamiltonian

$$H = \dot{X}.P - H = \frac{1}{2m}(P + eA)^2, \tag{3}$$

we need to remember which variables are canonical. This information is encoded in the Poisson bracket structure of the theory and, in the quantum theory, is transferred onto commutation relations between operators. The fact that X and P are canonical means that

$$\{x_i, p_j\} = \delta_{ij}, \qquad \{x_i, x_j\} = \{p_i, p_j\} = 0, \tag{4}$$

here, P is not gauge invariant. This means that the numerical value of P can't have any physical meaning since it depends on our choice of gauge. In contrast, the mechanical momentum $m\dot{X}$ is gauge invariant, it measures what you would physically call "momentum". But it doesn't have canonical Poisson structure.

2.2 Quantisation

Here, we are going to solve for the spectrum and wavefunctions of the quantum Hamiltonian, $H = \frac{1}{2m}(P + eA)^2$. Note that P and A are quantum operators. Since the particle is restricted to lie in the plane, we consider $X = (x, y)$. Also, we consider the magnetic field to be constant and perpendicular to this plane ($\nabla \times A = B\hat{z}$. The canonical commutation relations as follows

$$[x_i, p_j] = i\hbar\delta_{ij}, \qquad [x_i, x_j] = [p_i, p_j] = 0. \tag{5}$$

To find wavefunctions corresponding to the energy eigenstates, we first need to specify a gauge potential A such that ($\nabla \times A = B\hat{z}$. There is, of course, not a unique choice. Here, we work with the choice $A = xB\hat{y}$, this is called Landau gauge. Note that the magnetic field B is invariant under both translational symmetry and rotational symmetry in the (x, y)-plane. However, the choice of A is not, it breaks translational symmetry in the x direction (but not in the y direction) and rotational symmetry. This means that, while the physics will be invariant under all symmetries, the intermediate calculations will not be manifestly invariant. So, the Hamiltonian can be written as

$$H = \frac{1}{2m}(p_x^2 + (p_y + eBx)^2), \tag{6}$$

since we have translational invariance in the y direction, we can show that energy eigenstates are also eigenstates of p_y. These are just plane waves in the y direction. This motivates an ansatz using the separation of variables, $\psi_k(x, y) = \exp(iky) f_k(x)$. Acting on this wavefunction with the Hamiltonian, we see that the operator p_y just gets replaced by its eigenvalue $\hbar k$. On the other words, we have

$$H\psi_k(x, y) = \frac{1}{2m}(p_x^2 + (\hbar k + eBx)^2)\psi_k(x, y) \equiv H_k\psi_k(x, y), \tag{7}$$

now, the Hamiltonian reduce to the Hamiltonian for a harmonic oscillator in the x direction with the center displaced from the origin. Therefore, we have

$$H_k = \frac{1}{2m}p_x^2 + \frac{m\omega_B^2}{2}(x + kl_B^2)^2, \tag{8}$$

where $w_B = \frac{eB}{m}$ is the frequency of the harmonic oscillator. Also, $l_B = \sqrt{\frac{\hbar}{eB}}$ is a characteristic length scale which is called the magnetic length. For example, in a magnetic field of $B =$ 1 Tesla, the magnetic length for an electron is $l_B \approx 2.5 \times 10^{-8}m$. On the other hand, the momentum in the y direction, $\hbar k$, has turned into the position of the harmonic oscillator in the x direction, which is now centred at $x = -kl_B^2$. So, we can obtain the energy eigenvalues as

$$E_n = \hbar\omega_B(n + \frac{1}{2}), \tag{9}$$

Also, we can write down the explicit wavefunctions as

$$\psi_{n,k} \sim \exp(iky)H_n(x + kl_B^2)\exp\frac{-(x + kl_B^2)^2}{2l_B^2}, \tag{10}$$

where the wavefunctions depend on two quantum numbers, $n \in N$ and $k \in R$. Also, H_n are the usual Hermite polynomial wavefunctions of the harmonic oscillator. The $\sim$ reflects the fact that we have made no attempt to normalise these wavefunctions.

3 Effects of an electric field

One of the things that is particularly easy in the Landau gauge is the addition of an electric field $\overrightarrow{E}$ in the x direction. We can show this by the addition of an electric potential $\phi = -Ex$. So, the Hamiltonian become

$$H = \frac{1}{2m}(p_x^2 + (p_y + eBx)^2) - eEx, \tag{11}$$

then, we have

$$\left[\frac{1}{2m}P_X^2 + \frac{m\omega_B^2}{2}(X + kl_B^2)^2 + ekl_B^2E - \frac{e^2E^2}{2m\omega_B 2}\right]\psi_{n,k}(X,y) = E_{n,k}\psi_{n,k}(X,y), \tag{12}$$

where

$$\psi_{n,k}(X,y) = \psi_{n,k}(x - \frac{mE}{eB^2},y), \tag{13}$$

and the energies are as

$$E_{n,k} = \hbar\omega_B(n + \frac{1}{2}) + eE(kl_B^2 - \frac{eE}{m\omega_B^2}) + \frac{m}{2}\frac{E^2}{B^2}. \tag{14}$$

Now, the energy in each level depends linearly on k, so the degeneracy in each Landau level has been lifted. Because the energy depends on the momentum, it means that states drift in the y direction. The group velocity becomes

$$v_y = \frac{1}{\hbar}\frac{\partial E_{n,k}}{\partial k} = e\hbar El_B^2 = \frac{E}{B}. \tag{15}$$

Using equation (14), we obtain a natural interpretation which a wavepacket with momentum k is localised at position $x = -kl_B^2 + \frac{eE}{m\omega_B^2}$, its potential energy can be thought as $\phi = -Ex = eE(kl_B^2 - \frac{eE}{m\omega_B^2})$ and the kinetic energy for the particle can be as $\frac{1}{2}mv_y^2 = \frac{m}{2}\frac{E^2}{B^2}$.

4 Effect of Generalized Uncertainty Principle

As we know that Heisenberg obtained the Uncertainty Principle on very general grounds, using only the quantization of the electromagnetic radiation field. He did not consider gravitational effects in his uncertainty relation, as are usually assumed to be negligible. However, at increasingly large energies the gravitational interaction is more and more important. Various approaches to quantum gravity (such as string theory, doubly special relativity theories, as well as black hole physics) suggest that near the Planck scale, the Heisenberg Uncertainty Principle should be modified. The modified Uncertainty Principle is called Generalized Uncertainty Principle (GUP) [3,8,11]. These corrections are generically quite small to be measurable. However they could signal a new intermediate length scale between the electroweak and the Planck scale [7]. A generalized uncertainty principle (GUP) which is consistent with String theory, Doubly Special Relativity theories and Black hole Physics is [2]

$$[x_i, p_j] = i\hbar[1 + \beta p^2]\delta_{ij}, \tag{16}$$

where $[x_i, x_j] = 0$, $[p_i, p_j] = 0$ (via the Jacobi identity), $p^2 = p_i p^i$, $\beta = \beta_0 / M_{Pl}$, $M_{Pl}c^2 = Planck\ energy$ $= \frac{l_{Pl}}{\hbar} \approx 10^{19} GeV$, and $l_{Pl} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35} m$ Planck length. It is assumed that the dimensionless parameter β_0 is of the order of unity, in which case the β dependent terms are important only when energies (momentum) are comparable to the Planck energy (momentum), and lengths are comparable to the Planck length. The following definitions,

$$p_i = p_{0i}(1 + \beta p_0^2) \qquad\qquad x_i = x_{0i}, \tag{17}$$

satisfy the equation (16). p_{0i} is the canonical momentum $(p_{0i} = -i\hbar \frac{\partial}{\partial x_{0i}})$ and x_{0i}, p_{0i} satisfy the canonical commutation relations

$$[x_{0i}, p_{0i}] = i\hbar\delta_{ij} \qquad\qquad [x_{0i}, x_{0j}] = 0 \qquad\qquad [p_{0i}, p_{0j}] = 0. \tag{18}$$

Here, p_{0i} is the momentum at low energies and p_i (the modified momentum) is the momentum at higher energies [2]. Now, if we substitute relations (17) in equation (11), we obtain

$$\begin{aligned} H' &= \frac{1}{2m}\left[p_x + \beta p^2 p_x\right]^2 + \frac{1}{2m}\left[p_y + \beta p^2 p_y + eBx\right]^2 - eEx \\ &\simeq H + \frac{\beta}{m}\left[p^4 + eB(xp_y^3 + p_y x p_x^2 - i\hbar p_y p_x)\right], \end{aligned} \tag{19}$$

because β is very small parameter, we have neglected terms of the order of β^2. Also, using the perturbation theory, we can write $H' = H + \beta V$ and calculate the first order correction of the energy eigenvalues by $\Delta_{n,k}^{(1)} =< \psi_{n,k}|V|\psi_{n,k} >$. Then, using equation (14), we have $E'_{n,k} = E_{n,k} + \beta\Delta_{n,k}^{(1)}$.

Concluding remarks: In this paper, using the Landau gauge, we studied the energy levels of an electron moving in a thin film. This film was supposed as a two-dimensional electron gas which was under the influence of a uniform external magnetic field B and a uniform external electric field E. Here, the magnetic field was perpendicular to the film. We saw that a wavepacket with momentum k was localised at position $x = -kl_B^2 + \frac{eE}{m\omega_B^2}$. Also, the kinetic energy for the particle was found as $\frac{m}{2}\frac{E^2}{B^2}$. In fact, if we put an electric field E perpendicular to a magnetic field B, the cyclotron orbits of the electron drift. But, they don't drift in the direction of the electric field, instead they drift in the direction $E \times B$. Here, we see the quantum version of this statement. Finally, using the perturbation theory, we can study the effect of GUP on our model.

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