



## New version of degree-based topological indices of certain nanotube

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**Abstract.** In this paper, computation of the Arithmetic-Geometric index ( $AG_1$  index),  $SK$  index,  $SK_1$  index and  $SK_2$  index of H-Naphtalenic nanotube and  $TUC_4[m, n]$  nanotube. We also compute  $SK_3$  index,  $AG_2$  index for H-Naphtalenic nanotube and  $TUC_4[m, n]$  nanotube.

**Keywords.** Arithmetic-Geometric index ( $AG_1$  index),  $SK$  index,  $SK_1$  index,  $SK_2$  index,  $SK_3$  index,  $AG_2$  index, H-naphtalenic nanotube,  $TUC_4[m, n]$  nanotube.

### 1 Introduction

Let  $G$  be a simple connected graph in chemical graph theory. In mathematics chemistry, a molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. And also a connected graph is a graph such that there is a path between all pairs of vertices. Note that hydrogen atoms are often omitted [5].

Mathematical chemistry is a branch of theoretical chemistry for discussion and prediction of the molecular structure using mathematical methods without necessarily referring to quantum mechanics. Chemical graph theory is a branch of mathematical chemistry which applies graph theory to mathematical modelling of chemical phenomena [8, 17, 21, 22]. This theory had an important effect on the development of the chemical sciences.

All molecular graphs considered in this paper are finite, connected, loopless, and without multiple edges. Let  $G = (V, E)$  be a graph with  $n$  vertices and  $m$  edges. The degree of a vertex

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$u \in V(G)$  is denoted by  $d_G(u)$  and is the number of vertices that are adjacent to  $u$ . The edge connecting the vertices  $u$  and  $v$  is denoted by  $uv$  [9].

## 2 Computing the topological indices of certain nanotube

Motivated by previous research on certain nanotube, here we introduce six new topological indices and computed their corresponding topological index value of certain nanotube [10, 11, 13, 18–20].

Carbon Nanotubes, long, thin cylinders of carbon, were discovered in 1991 by S. Iijima. These are large macromolecules that are unique for their size, shape, and remarkable physical properties. They can be thought of as a sheet of graphite (a hexagonal lattice of carbon) rolled into a cylinder. These intriguing structures have sparked much excitement in recent years and a large amount of research has been dedicated to their understanding. Currently, the physical properties are still being discovered and disputed. Nanotubes have a very broad range of electronic, thermal, and structural properties that change depending on the different kinds of nanotube (defined by its diameter, length, and chirality, or twist). To make things more interesting, besides having a single cylindrical wall (SWNTs), Nanotubes can have multiple walls (MWNTs)–cylinders inside the other cylinders. For recent work on computing topological indices of certain nanotube can be seen in [10, 11, 14].

The distance between two vertices  $a$  and  $b$  is denoted as  $d_H(a, b)$  and is the length of shortest path between  $a$  and  $b$  in graph  $H$ . The length of shortest path between  $a$  and  $b$  is also called  $a$ - $b$  geodesic. The longest path between any two vertices is called  $a$ - $b$  detour.

In this paper,  $H$  is considered to be simple connected graph with vertex set  $V(H)$  and edge set  $E(H)$ ,  $d_a$  is the degree of vertex  $a \in V(H)$  and

$$S_a = \sum_{b \in N_H(a)} d(b),$$

where  $N_H = \{b \in V(H) \mid ab \in E(H)\}$ . The notations used in this paper are mainly taken books [4, 21]. Now, we propose the following topological indices and compute their value for certain nanotube.

### 2.1 Arithmetic-Geometric ( $AG_1$ ) Index

Let  $G = (V, E)$  be a molecular graph, and  $d_u$  is the degree of the vertex  $u$ . Then  $AG_1$  index of  $G$  is defined as

$$AG_1(G) = \sum_{u, v \in E(G)} \frac{d_G(u) + d_G(v)}{2\sqrt{d_G(u) \cdot d_G(v)'}}$$

Where  $AG_1$  index is considered for distinct vertices. The above equation is the sum of the ratio of the Arithmetic mean and Geometric mean of  $u$  and  $v$ , where  $d_G(u)$  (or  $d_G(v)$ ) denotes the degree of the vertex  $u$  (or  $v$ ).

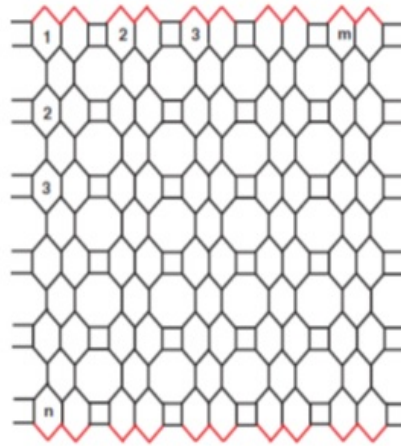


Figure 1. A 2D-lattice of H-Naphtalenic nanotube  $NPHX[m, n]$  showing the edge partition based on the degrees of end vertices of each edge.

## 2.2 SK index

The SK index of a graph  $G = (V, E)$  is defined as

$$SK(G) = \sum_{u,v \in E(G)} \frac{d_G(u) + d_G(v)}{2},$$

where  $d_G(u)$  and  $d_G(v)$  are the degrees of the vertices  $u$  and  $v$  in  $G$ .

## 2.3 $SK_1$ index

The  $SK_1$  index of a graph  $G = (V, E)$  is defined as

$$SK_1(G) = \sum_{u,v \in E(G)} \frac{d_G(u) \cdot d_G(v)}{2},$$

where  $d_G(u)$  and  $d_G(v)$  are the product of the degrees of the vertices  $u$  and  $v$  in  $G$ .

## 2.4 $SK_2$ index

The  $SK_2$  index of a graph  $G = (V, E)$  is defined as

$$SK_2(G) = \sum_{u,v \in E(G)} \left( \frac{d_G(u) + d_G(v)}{2} \right)^2,$$

where  $d_G(u)$  and  $d_G(v)$  are the product of the degrees of the vertices  $u$  and  $v$  in  $G$ .

## 2.5 $SK_3$ index

The  $SK_3$  index of a graph  $G = (V, E)$  is defined as

$$SK_3(G) = \sum_{u,v \in E(G)} \frac{S_G(u) + S_G(v)}{2},$$

where  $S_G(u)$  and  $S_G(v)$  is the summation of degrees of all neighbours of vertices  $u$  and  $v$  in  $G$ .

$$S_G(u) = \sum_{u,v \in E(G)} d_G(u),$$

$$N_G(u) = \{v \in V(G) | uv \in E(G)\}.$$

## 2.6 Arithmetic-Geometric ( $AG_2$ ) index

Let  $G = (V, E)$  be a molecular graph, and  $S_u$  is the degree of the vertex  $u$ , then  $AG_2$  index of  $G$  is defined as

$$AG_2(G) = \sum_{u,v \in E(G)} \frac{S_G(u) + S_G(v)}{2\sqrt{S_G(u) \cdot S_G(v)}}$$

Where  $AG_2$  index is considered for distinct vertices. The above equation is the sum of the ratio of the Arithmetic mean and Geometric mean of  $u$  and  $v$ , where  $S_G(u)$  (or  $S_G(v)$ ) is the summation of degrees of all neighbours of vertex  $u$  (or  $v$ ).

$$S_G(u) = \sum_{u,v \in E(G)} d_G(u),$$

$$N_G(u) = \{v \in V(G) / uv \in E(G)\}.$$

In this paper, we study certain degree based topological indices of H-Naphtalenic nanotubes and  $TUC_4[m, n]$  nanotube. These topological indices correlate certain physico-chemical properties of these nanotube.

## 3 Main results

In this paper, we study  $AG_1$ ,  $SK$ ,  $SK_1$ ,  $SK_2$ ,  $SK_3$  and  $AG_2$  indices of H-Naphtalenic Nanotube,  $TUC_4[m, n]$  Nanotube. In this section, we compute these topological indices for H-Naphtalenic nanotubes.

### 3.1 Results for H-Naphtalenic nanotubes

In this section, we compute the certain topological indices for H-Naphtalenic nanotubes. This nanotube is a trivalent decoration having sequence of  $C_6, C_6, C_4, C_6, C_6, C_4$ , in first row and a sequence of  $C_6, C_8, C_6, C_8$ , in other row. In other words, the whole lattice is a plane tiling can either cover a cylinder or a torus. These nanotubes usually symbolized as  $NPHX[m, n]$  in which  $m$  is the number of pair of hexagons in first row and  $n$  is the number of alternative hexagons in a column as depicted in Figure 1. Now we compute certain degree based topological indices for this class of nanotubes. We can clearly see that, there are two type of edges in 2D-lattice of this nanotube as shown by different colours in Figure 1, in which red colour shows the edges  $ab$  and  $d_a = 2$  and  $d_b = 3$  and black colour shows the edges  $ab$  with  $d_a = d_b = 3$ . Table 1 shows cardinalities of these two partite sets of edge set of  $NPHX[m, n]$  nanotube.

$(d_a, d_b)$ where $a, b \in E(H)$	(2,3)	(3,3)
Number of edges	8m	15mn – 10m

Table 1. Edge partition of 2D-lattice of H-Naphthalenic nanotubes based on degrees of end vertices of each edge.

**Theorem 3.1.** Consider the graph of  $NPHX[m, n]$  nanotubes, then its  $AG_1$  index is equal to

$$AG_1(NPHX[m, n]) = (15n + 20 - 10\sqrt{6})m.$$

*Proof.* Consider the graph of  $NPHX[m, n]$ . By using the edge partition based on the degrees of end vertices of each edge of graph of  $NPHX[m, n]$  nanotube given in Table 1, we compute the  $AG_1$  index of  $NPHX[m, n]$  nanotube.

$$AG_1(G) = \sum_{u,v \in E(G)} \frac{d_G(u) + d_G(v)}{2\sqrt{d_G(u) \cdot d_G(v)}}.$$

$$\begin{aligned} AG_1(NPHX(n)) &= 8m \left( \frac{2+3}{2\sqrt{6}} \right) + (15mn - 10m) \left( \frac{3+3}{2\sqrt{9}} \right) \\ &= \frac{20m}{\sqrt{6}} + 15mn - 10m \\ &= 15mn + \left( \frac{20}{\sqrt{6}} - 10 \right) m \\ &= (15n + 20 - 10\sqrt{6}) m. \end{aligned}$$

□

**Theorem 3.2.** Consider the graph of  $NPHX[m, n]$  nanotubes, then its  $SK$  index is equal to

$$SK(NPHX[m, n]) = (45n - 10)m.$$

*Proof.* Consider the graph of  $NPHX[m, n]$ . By using the edge partition based on the degrees of end vertices of each edge of graph of  $NPHX[m, n]$  nanotube given in Table 1, we compute the  $SK$  index of  $NPHX[m, n]$  nanotube.

$$SK(G) = \sum_{u,v \in E(G)} \frac{d_G(u) + d_G(v)}{2}.$$

$$\begin{aligned} SK(NPHX(n)) &= 8m \left( \frac{2+3}{2} \right) + (15mn - 10m) \left( \frac{3+3}{2} \right) \\ &= 20m + 45mn - 30m \\ &= 45mn - 10m \\ &= (45n - 10) m. \end{aligned}$$

□

**Theorem 3.3.** Consider the graph of  $NPHX[m, n]$  nanotubes, then its  $SK_1$  index is equal to

$$SK_1(NPHX[m, n]) = \frac{1}{2} (135n - 42) m.$$

*Proof.* Consider the graph of  $NPHX[m, n]$ . By using the edge partition based on the degrees of end vertices of each edge of graph of  $NPHX[m, n]$  nanotube given in Table 1, we compute the  $SK_1$  index of  $NPHX[m, n]$  nanotube.

$$SK_1(G) = \sum_{u,v \in E(G)} \frac{d_G(u) \cdot d_G(v)}{2}.$$

$$\begin{aligned} SK_1(NPHX(n)) &= 8m \left( \frac{2 \times 3}{2} \right) + (15mn - 10m) \left( \frac{3 \times 3}{2} \right) \\ &= 24m + (15mn - 10m) \frac{9}{2} \\ &= \frac{1}{2} (48m + 135mn - 90m) \\ &= \frac{1}{2} (135n - 42) m. \end{aligned}$$

□

**Theorem 3.4.** Consider the graph of  $NPHX[m, n]$  nanotubes, then its  $SK_2$  index is equal to

$$SK_2(NPHX[m, n]) = (135n - 40) m.$$

*Proof.* Consider the graph of  $NPHX[m, n]$ . By using the edge partition based on the degrees of end vertices of each edge of graph of  $NPHX[m, n]$  nanotube given in Table 1, we compute the  $SK_2$  index of  $NPHX[m, n]$  nanotube.

$$SK_2(G) = \sum_{u,v \in E(G)} \left( \frac{d_G(u) + d_G(v)}{2} \right)^2.$$

$$\begin{aligned} SK_2(NPHX(n)) &= 8m \left( \frac{2+3}{2} \right)^2 + (15mn - 10m) \left( \frac{3+3}{2} \right)^2 \\ &= 50m + 135mn - 90m \\ &= (135n - 40) m. \end{aligned}$$

Now we compute two important topological indices  $AG_2$  and  $SK_3$  for 2D-lattice of  $NHPX[m, n]$  nanotube. There are six types of edges in  $NHPX[m, n]$  nanotube based on the degree sum of vertices lying at unit distance from end vertices of each edge as depicted in Figure 2, in which different colours shows different partite sets of edge set of  $NHPX[m, n]$

$(S_a, S_b)$ where $u, v \in E(H)$	(6,7)	(6,8)	(8,8)	(7,9)	(8,9)	(9,9)
Number of edges	$4m$	$4m$	$2m$	$2m$	$4m$	$15mn - 18m$

Table 2. Edge partition of graph of  $NHPX[m, n]$  nanotubes based on degree sum of vertices lying at unit distance from end vertices of each edge.

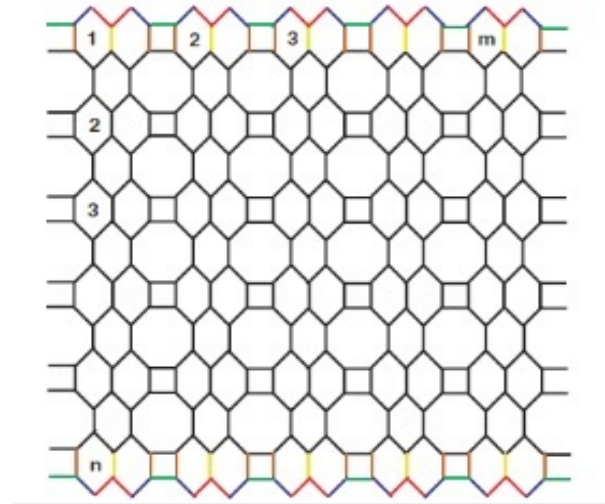


Figure 2. A graph of H-Naphthalenic nanotube  $NPHX[m, n]$  showing different partite sets based on the degree sum of neighbours of end vertices of each edge.

nanotube. In Figure 2, red colour shows the edges  $ab$  with  $S_a = 6$  and  $S_b = 7$ , blue colour shows the type of edges  $ab$  with  $S_a = 6$  and  $S_b = 8$ , green colour shows the type of edges  $ab$  with  $S_a = S_b = 8$ , yellow colour shows the type of edges  $ab$  with  $S_a = 7$  and  $S_b = 9$ , brown colour shows the type of edges  $ab$  with  $S_a = 8$  and  $S_b = 9$  and black colour shows the partition having edges  $ab$  with  $S_a = S_b = 9$ .

In Table 2, cardinalities of such partite sets of edge set of graph of  $NPHX[m, n]$  nanotube are shown. In the following theorem  $AG_2$  index of  $NPHX[m, n]$  nanotube and  $SK_3$  index of  $NPHX[m, n]$  nanotube is computed.  $\square$

**Theorem 3.5.** Consider the graph of  $NPHX[m, n]$  nanotubes, then its  $AG_2$  index is equal to

$$AG_2(NPHX[m, n]) = \left( 15n + \frac{26}{\sqrt{42}} + \frac{28}{\sqrt{48}} + \frac{16}{\sqrt{63}} + \frac{34}{\sqrt{72}} - 16 \right) m.$$

*Proof.* We use the edge partition of graph of  $NPHX[m, n]$  nanotube based on the degree sum of vertices lying at unit distance from end vertices of each edge. Now by using the partition given in Table 2 we can apply the formula of  $AG_2$  index to compute this index for  $NPHX[m, n]$  nanotube.

$$AG_2(G) = \sum_{u, v \in E(G)} \frac{S_G(u) + S_G(v)}{2\sqrt{S_G(u) \cdot S_G(v)}}.$$

$$\begin{aligned}
 AG_2(NPHX(n)) &= 4m \left( \frac{6+7}{2\sqrt{42}} \right) + 4m \left( \frac{6+8}{2\sqrt{48}} \right) + 2m \left( \frac{8+8}{2\sqrt{64}} \right) + 2m \left( \frac{7+9}{2\sqrt{63}} \right) \\
 &\quad + 4m \left( \frac{8+9}{2\sqrt{72}} \right) + (15mn - 8m) \left( \frac{9+9}{2\sqrt{81}} \right) \\
 &= \frac{26m}{\sqrt{42}} + \frac{28m}{\sqrt{48}} + 2m + \frac{16m}{\sqrt{63}} + \frac{34m}{\sqrt{72}} + 15mn - 18m \\
 &= \left( 15n + \frac{26}{\sqrt{42}} + \frac{28}{\sqrt{48}} + \frac{16}{\sqrt{63}} + \frac{34}{\sqrt{72}} - 16 \right).
 \end{aligned}$$

□

**Theorem 3.6.** Consider the graph of  $NPHX[m, n]$  nanotubes, then its  $SK_3$  index is equal to

$$SK_3(NPHX[m, n]) = (135n - 42)m.$$

*Proof.* We use the edge partition of graph of  $NPHX[m, n]$  nanotube based on the degree sum of vertices lying at unit distance from end vertices of each edge. Now by using the partition given in Table 2 we can apply the formula of  $SK_3$  index to compute this index for  $NPHX[m, n]$  nanotube.

$$SK_3(G) = \sum_{u,v \in E(G)} \frac{S_G(u) + S_G(v)}{2}.$$

$$\begin{aligned}
 SK_3(NPHX(n)) &= 4m \left( \frac{6+7}{2} \right) + 4m \left( \frac{6+8}{2} \right) + 2m \left( \frac{8+8}{2} \right) + 2m \left( \frac{7+9}{2} \right) + 4m \left( \frac{8+9}{2} \right) \\
 &\quad + (15mn - 8m) \left( \frac{9+9}{2} \right) \\
 &= 26m + 28m + 16m + 34m + 135mn - 162m \\
 &= (135n - 42)m.
 \end{aligned}$$

### 3.2 Results for nanotubes covered by $C_4$

In this section, we compute certain topological indices of nanotube covered only by  $C_4$ . The 2D-lattice of this family of nanotubes is a plane tiling of  $C_4$ . This tessellation of  $C_4$  can either cover a cylinder or a torus. This family of nanotubes is denoted by  $TUC_4[m, n]$ , in which  $m$  is the number of squares in a row and  $n$  is the number of squares in a column as shown in Figure 4. A 3D representation of  $TUC_4[m, n]$  nanotubes is depicted in Figure 3. There are three types of edges in 2D-lattice of  $TUC_4[m, n]$  nanotube based on degrees of end vertices of each edge. Figure 3, explains such a partition of edges in which red coloured edges are the edges  $ab$  with  $d_a = d_b = 3$ , blue coloured edges are the edges  $ab$  with  $d_a = 3, d_b = 4$  and black coloured edges are the edges  $ab$  with  $d_a = d_b = 4$ . Table3 shows the number of edged in each partite set. □

**Theorem 3.7.** Consider the graph of  $TUC_4[m, n]$  nanotubes, then its  $AG_1$  index is equal to

$$AG_1(TUC_4[m, n]) = \left( 2n + \frac{7}{\sqrt{12}} - 1 \right) (m + 1).$$



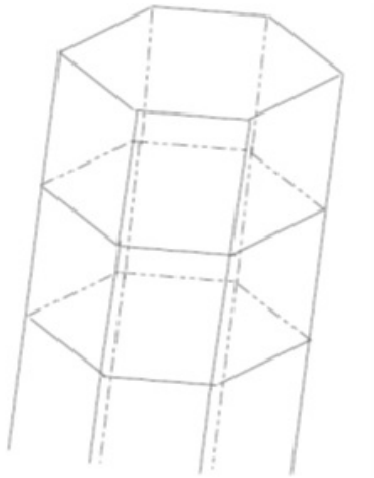


Figure 3. A  $TUC_4[6, n]$  nanotube covered by  $C_4$ .

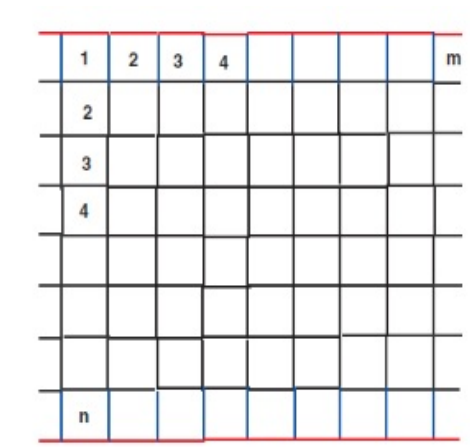


Figure 4. A graph of  $TUC_4[m, n]$  nanotube showing the edge partition based on the degree of end vertices of each edge.

*Proof.* Consider the graph of  $TUC_4[m, n]$ . By using the edge partition based on the degrees of end vertices of each edge of graph of  $TUC_4[m, n]$  nanotube given in Table 3, we compute the  $AG_1$  index of  $TUC_4[m, n]$  nanotube.

$$AG_1(G) = \sum_{u,v \in E(G)} \frac{d_G(u) + d_G(v)}{2\sqrt{d_G(u) \cdot d_G(v)}}$$

$$\begin{aligned} AG_1(TUC_4[m, n]) &= (2m + 2) \left( \frac{3 + 3}{2\sqrt{9}} \right) + (2m + 2) \left( \frac{3 + 4}{2\sqrt{12}} \right) \\ &+ (m + 1)(2n - 3) \left( \frac{4 + 4}{2\sqrt{16}} \right) \\ &= (2m + 2) + (2m + 2) \left( \frac{7}{2\sqrt{12}} \right) + (m + 1)(2n - 3) \end{aligned}$$

$(d_a, d_b)$ where $u, v \in E(H)$	(3,3)	(3,4)	(4,4)
Number of edges	$(2m + 2)$	$(2m + 2)$	$(m + 1)(2n - 3)$

Table 3. Cardinalities of different partite sets based on degrees of end vertices of each edge of graph of  $TUC_4[m, n]$  nanotube.

$$\begin{aligned}
 &= 2mn + \left(\frac{7}{2\sqrt{12}}\right) - m + \left(\frac{7}{2\sqrt{12}}\right) - 1 \\
 &= \left(2n + \frac{7}{\sqrt{12}} - 1\right) - m + \frac{7}{12} - 1 + 2n \\
 &= \left(2n + \frac{7}{\sqrt{12}} - 1\right) (m + 1).
 \end{aligned}$$

□

**Theorem 3.8.** Consider the graph of  $TUC_4[m, n]$  nanotubes, then its SK index is equal to

$$SK(TUC_4[m, n]) = (8n + 1)(m + 1).$$

*Proof.* Consider the graph of  $TUC_4[m, n]$ . By using the edge partition based on the degrees of end vertices of each edge of graph of  $TUC_4[m, n]$  nanotube given in Table 3, we compute the SK index of  $TUC_4[m, n]$  nanotube.

$$SK(G) = \sum_{u,v \in E(G)} \frac{d_G(u) + d_G(v)}{2}.$$

$$\begin{aligned}
 SK(TUC_4[m, n]) &= (2m + 2) \left(\frac{3+3}{2}\right) + (2m + 2) \left(\frac{3+4}{2}\right) + (m + 1)(2n - 3) \left(\frac{4+4}{2}\right) \\
 &= 6m + 6 + 7m + 7 + (2mn - 3m + 2n - 3)4 \\
 &= (8n + 1)m + 8n + 1 \\
 &= (8n + 1)(m + 1).
 \end{aligned}$$

□

**Theorem 3.9.** Consider the graph of  $TUC_4[m, n]$  nanotubes, then its  $SK_1$  index is equal to

$$SK_1(TUC_4[m, n]) = (16n - 3)(m + 1).$$

*Proof.* Consider the graph of  $TUC_4[m, n]$ . By using the edge partition based on the degrees of end vertices of each edge of graph of  $TUC_4[m, n]$  nanotube given in Table 3, we compute the  $SK_1$  index of  $TUC_4[m, n]$  nanotube.

$$SK_1(G) = \sum_{u,v \in E(G)} \frac{d_G(u).d_G(v)}{2}.$$

$$\begin{aligned}
 SK_1(TUC_4[m,n]) &= (2m+2) \binom{3 \times 3}{2} + (2m+2) \binom{3 \times 4}{2} + (m+1)(2n-3) \binom{4 \times 4}{2} \\
 &= 9m+9+12m+12+(2mn-3m+2n-3)8 \\
 &= (16n-3)m+16n+1 \\
 &= (16n-3)(m+1).
 \end{aligned}$$

□

**Theorem 3.10.** Consider the graph of  $TUC_4[m,n]$  nanotubes, then its  $SK_2$  index is equal to

$$SK_2(TUC_4[m,n]) = \frac{1}{2}(64n-11)(m+1).$$

*Proof.* Consider the graph of  $TUC_4[m,n]$ . By using the edge partition based on the degrees of end vertices of each edge of graph of  $TUC_4[m,n]$  nanotube given in Table 3, we compute the  $SK_2$  index of  $TUC_4[m,n]$  nanotube.

$$SK_2(G) = \sum_{u,v \in E(G)} \left( \frac{d_G(u) + d_G(v)}{2} \right)^2.$$

$$\begin{aligned}
 SK_2(TUC_4[m,n]) &= (2m+2) \binom{3+3}{2}^2 + (2m+2) \binom{3+4}{2}^2 + (m+1)(2n-3) \binom{4+4}{2}^2 \\
 &= (2m+2)9 + (2m+2) \binom{49}{4} + (2mn-3m+2n-3)16 \\
 &= \frac{1}{2}(64mn-11m+64n-11) \\
 &= \frac{1}{2}((64n-11)m+64n-11) \\
 &= \frac{1}{2}(64n-11)(m+1).
 \end{aligned}$$

Now we compute  $AG_2$  and  $SK_3$  indices for two dimensional lattice of  $TUC_4[m,n]$  nanotubes. There are five types of edges in the graph of  $TUC_4[m,n]$  nanotube based on degree sum of vertices lying at unit distance from end vertices of each as shown in Figure 5 in which red coloured edges are the edge  $ab$  with  $S_a = S_b = 7$ , blue coloured edges are the edge  $ab$  with  $S_a = 7$  and  $S_b = 15$ , green coloured edges are the edge  $ab$  with  $S_a = S_b = 15$ , yellow coloured edges are the edge  $ab$  with  $S_a = 15$  and  $S_b = 16$ , black coloured edges are the edge  $ab$  with  $S_a = S_b = 16$ . Table 4 shows the cardinalities of these partite sets.

□

**Theorem 3.11.** Let the graph of  $TUC_4[m,n]$  nanotube with  $(m \geq 1, n \geq 4)$ , then its  $AG_2$  index is

$$AG_2(TUC_4[m,n]) = \left( 2n-3 + \frac{22}{\sqrt{105}} + \frac{31}{\sqrt{240}} \right) (m+1).$$

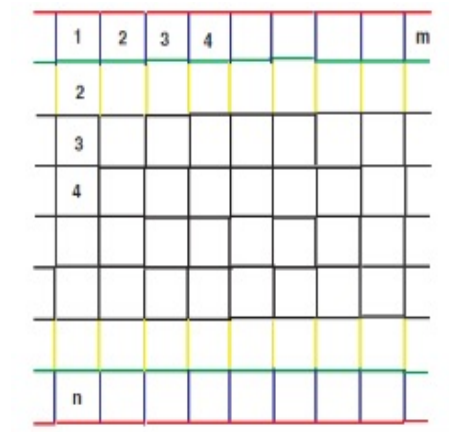


Figure 5. A graph of  $TUC_4[m, n]$  nanotube showing the edge partition based on the degree sum of end vertices lying at unit distance from end vertices of each edge.

Table 4. Edge partition of graph of  $TUC_4[m, n]$  nanotube based on degree sum of vertices lying at unit distance from end vertices of each edge.

$(S_a, S_b)$ where $u, v \in E(H)$	(7,7)	(7,15)	(15,15)	(15,16)	(16,16)
Number of edges	$(2m + 2)$	$(2m + 2)$	$(2m + 2)$	$(2m + 2)$	$(m + 1)(2n - 7)$

*Proof.* We use the edge partition of graph of  $TUC_4[m, n]$  nanotube based on the degree sum of vertices lying at unit distance from end vertices of each edge.

Now by using the partition given in Table 4 we can apply the formula of  $AG_2$  index to compute this index for  $TUC_4[m, n]$  nanotube.

$$\begin{aligned}
 AG_2(G) &= \sum_{u,v \in E(G)} \frac{S_G(u) + S_G(v)}{2\sqrt{S_G(u) \cdot S_G(v)}} \\
 AG_2(TUC_4[m, n]) &= (2m + 2) \left( \frac{7 + 7}{2\sqrt{49}} \right) + (2m + 2) \left( \frac{7 + 15}{2\sqrt{105}} \right) + (2m + 2) \left( \frac{15 + 15}{2\sqrt{225}} \right) \\
 &+ (2m + 2) \left( \frac{15 + 16}{2\sqrt{240}} \right) + (m + 1)(2n - 7) \left( \frac{16 + 16}{2\sqrt{256}} \right) \\
 &= 2m + 2 + \frac{22m + 22}{\sqrt{105}} + 2m + 2 + \frac{31m + 31}{\sqrt{240}} + 2mn - 7m + 2n - 7 \\
 &= 2mn - 3m + 2n - 3 + \frac{22m + 22}{\sqrt{105}} + \frac{31m + 31}{\sqrt{240}} \\
 &= \left( 2n - 3 + \frac{22}{\sqrt{105}} + \frac{31}{\sqrt{240}} \right) m + \left( 2n - 3 + \frac{22}{\sqrt{105}} + \frac{31}{\sqrt{240}} \right) \\
 &= \left( 2n - 3 + \frac{22}{\sqrt{105}} + \frac{31}{\sqrt{240}} \right) (m + 1).
 \end{aligned}$$

□

**Theorem 3.12.** Let the graph of  $TUC_4[m, n]$  nanotube with  $(m \geq 1, n \geq 4)$ , then its  $SK_3$  index is

$$SK_3(TUC_4[m, n]) = (32n - 15)(m + 1).$$

*Proof.* We use the edge partition of graph of  $TUC_4[m, n]$  nanotube based on the degree sum of vertices lying at unit distance from end vertices of each edge. Now by using the partition given in Table 4 we can apply the formula of  $AG_2$  index to compute this index for  $TUC_4[m, n]$  nanotube.

$$SK_3(G) = \sum_{u,v \in E(G)} \frac{S_G(u) + S_G(v)}{2}.$$

$$\begin{aligned} SK_3(TUC_4[m, n]) &= (2m + 2) \left( \frac{7 + 7}{2} \right) + (2m + 2) \left( \frac{7 + 15}{2} \right) + (2m + 2) \left( \frac{15 + 15}{2} \right) \\ &+ (2m + 2) \left( \frac{15 + 16}{2} \right) + (m + 1)(2n - 7) \left( \frac{16 + 16}{2} \right) \\ &= 14m + 14 + 22m + 22 + 30m + 30 + 31 + (2mn - 7m + 2n - 7)16 \\ &= 32mn - 15m + 32n - 15 \\ &= (32n - 15)m + 32n - 15 \\ &= (32n - 15)(m + 1). \end{aligned}$$

□

**Concluding remarks:** A generalized formula for Arithmetic-Geometric index ( $AG_1$  index),  $SK$  index,  $SK_1$  index,  $SK_2$  index,  $SK_3$  index,  $AG_2$  index for H-Naphtalenic nanotube and  $TUC_4[m, n]$  nanotube is obtained without using computer.

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