

## Computing the energy of small fullerenes

TAYYEBEH GHORBANI

*Institute of Nanoscience and Nanotechnology, University of Kashan,*

*8731751167 Kashan, I. R Iran*

*Email: ghorbani.tayebah@gmail.com*

**ABSTRACT.** The concept of energy of graph is defined in 1978 by Ivan Gutman as the sum of the absolute values of the eigenvalues of a graph. In this article we estimate the energy of small fullerenes.

**Keywords:** eigenvalue, fullerene, graph energy.

### 1. INTRODUCTION

In graph theory a **fullerene** is a three connected cubic graphs with pentagonal and hexagonal faces satisfying in Euler's formula. In 1985, a team of scientists discovered a previously unknown pure carbon molecule, namely the most stable fullerene  $C_{60}$ , which they called buckminsterfullerene, see [6,7].

By using Euler's theorem, one can easily prove that a fullerene on  $n$  vertices has exactly 12 pentagons and  $n/2 - 10$  hexagons, while  $n$  is a natural number equal or greater than 20 and  $n \neq 22$ .

Let  $G$  be a simple molecular graph namely a graph without directed and multiple edges and without loops. The vertex and edge-sets of  $G$  are represented by  $V(G)$  and  $E(G)$ , respectively. The adjacency matrix  $A(G)$  of graph  $G$  with vertex set  $V(G) = \{v_1, v_2, \dots, v_n\}$  is the  $n \times n$  symmetric matrix  $[a_{ij}]$  such that  $a_{ij} = 1$  if  $v_i$  and  $v_j$  are adjacent and 0, otherwise. The characteristic polynomial  $\chi(G, \lambda)$  of graph  $G$  is defined as

$$\chi(G, \lambda) = \det(A(G) - \lambda I).$$

The roots of this polynomial are eigenvalues of  $G$  and form the spectrum of graph as follows:

$$\text{Spec}(G) = \begin{pmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ t_1 & t_2 & \cdots & t_n \end{pmatrix}$$

where  $t_i$  is the multiplicity of eigenvalue  $\lambda_i$ . If  $G$  is a graph on  $n$  vertices and  $\lambda_1, \lambda_2, \dots, \lambda_n$  are the eigenvalues of its adjacency matrix, then the energy of  $G$  is defined as

$$E(G) = \sum_{i=1}^n |\lambda_i|.$$

This graph invariant is very closely related to a chemical quantity known as the total  $\pi$ -electron energy of conjugated hydrocarbon molecules, see [1,2,4,5].

## 2. Results and Discussion

Assume that  $F$  is a fullerene with at most 50 vertices. The energy of all isomers of fullerene  $F$  is reported in Appendix.

**Theorem 1 [3] (Interlacing theorem).** Let  $A$  be a real symmetric matrix with eigenvalues  $\lambda_1 \geq \dots \geq \lambda_n$  and let  $\mu_1 \geq \dots \geq \mu_m$  be the eigenvalues of a principal submatrix of  $A$ . Then for  $i=1, 2, \dots, n-1$ , we have

$$\lambda_i \geq \mu_i \geq \lambda_{n-m+i}.$$

**Corollary 2.** Let  $G$  be a graph with eigenvalues  $\lambda_1 \geq \dots \geq \lambda_n$ , and  $H$  be an induced subgraph of  $G$  with eigenvalues  $\mu_1 \geq \dots \geq \mu_m$ . Then for  $i=1, 2, \dots, n-1$ , we have

$$\lambda_i \geq \mu_i \geq \lambda_{n-m+i}.$$

**Theorem 3.** Let  $F_i$  ( $1 \leq i \leq 20$ ) be an induced subgraph of fullerene  $F$  with eigenvalues  $\theta_1, \dots, \theta_{m_i}$  and let  $\lambda_1, \dots, \lambda_n$  be eigenvalues of  $F$ , then

$$E(F) \geq (3 - \theta_1) + \frac{1}{2}E(F_i) + (n - m_i - 1)|\theta_{r_i+1}|$$

where  $r_i$  is the number of positive eigenvalues of graph  $F_i$  with  $m_i$  vertices.

**Proof.** Let  $r_i$  is the number of positive eigenvalues of graph  $F_i$  with  $m_i$  vertices, then by using interlacing theorem we have

$$\begin{aligned} E(F) &= \sum_{i=1}^n |\lambda_i| = 3 + \sum_{i=2}^{r_i} |\lambda_i| + \sum_{i=r_i+1}^n |\lambda_i| \geq 3 + \sum_{i=2}^{r_i} |\theta_i| + \sum_{i=r_i+1}^n |\lambda_i| \\ &\geq (3 - \theta_1) + |\lambda_{r_i+1}| + \frac{1}{2}E(F_i) + (n - m_i - 1)|\theta_{r_i+1}|. \end{aligned}$$

Here, by using interlacing theorem, we can introduce a lower bound for fullerene graphs with respect to induced subgraphs  $F_1$ - $F_{20}$  as depicted in Figure 1. To do this, let  $F$  be a fullerene graph on  $n$  vertices. It is clear that one of subgraphs  $F_1$ - $F_{20}$  is an induced subgraph of  $F$ . By using these values, we can verify lower bounds for the energy of related fullerenes. These values are reported in Table 1.

$F_i$	$r_i$	$\theta_1$	$ \theta_{r_i+1} $	$E(F_i)$	$m_i$	Lower Bounds
$F_1$	8	2.7616	0.3402	21.47	15	$0.34n+5.9$
$F_2$	10	2.7081	0.7630	28.74	20	$0.76n-60$
$F_3$	10	2.7616	0.4450	23.30	17	$0.45n+4.3$
$F_4$	8	2.7439	0.0337	22.63	16	$0.03n+11$
$F_5$	9	2.4196	0.0400	24.09	17	$0.04n+12$
$F_6$	10	2.7253	0.9455	24.04	17	$0.95n-3.8$
$F_7$	10	2.6751	0.4773	25.77	18	$0.78n+4.6$
$F_8$	10	2.7103	0.7934	25.69	18	$0.79n-1.1$
$F_9$	10	2.6921	0.6021	27.29	19	$0.6n+2.5$
$F_{10}$	10	2.7318	0.0579	28.58	20	$0.06n+13$
$F_{11}$	10	2.7338	0.1953	28.69	20	$0.2n+10.7$
$F_{12}$	11	2.7313	0.3111	28.55	20	$0.31n+8.3$
$F_{13}$	12	2.7159	0.8952	29.78	21	$0.9n-3.63$
$F_{14}$	11	2.7176	0.1268	30.01	21	$0.13n+12.6$
$F_{15}$	11	2.7201	0.1494	29.96	21	$0.15n+12.1$
$F_{16}$	12	2.7027	0.8019	31.51	22	$0.8n-1.6$
$F_{17}$	12	2.7030	0.7945	31.47	22	$0.79n-1.44$
$F_{18}$	12	2.7051	0.7611	31.41	22	$0.76n-0.74$
$F_{19}$	12	2.6895	0.6653	33.07	23	$0.66n+1.54$
$F_{20}$	12	2.6751	0.5392	34.57	24	$0.54n+4.67$

**Table 1.** The lower bound for the energy of fullerene graph  $F$  when  $F_i \leq F$ .

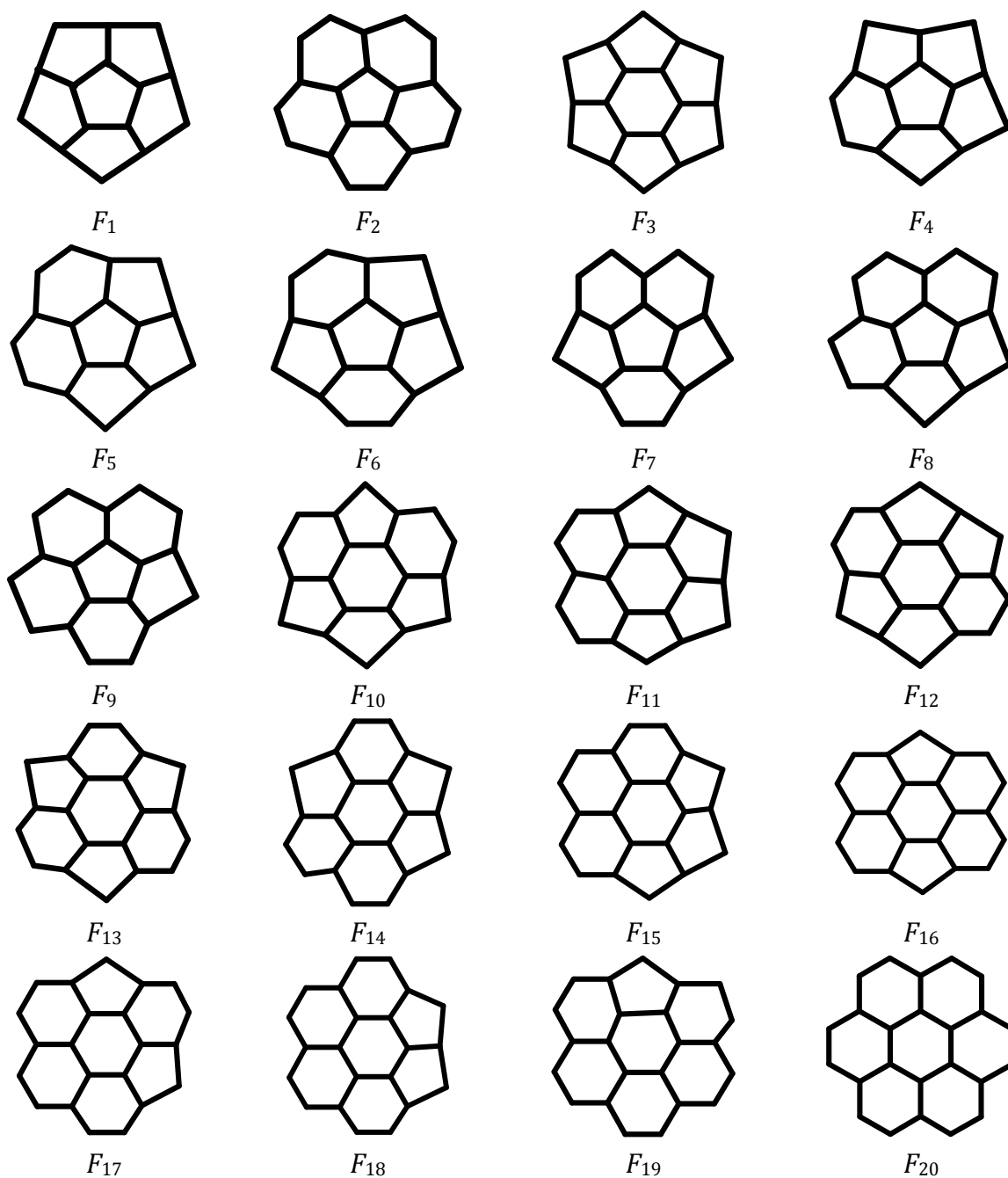


Figure 1. Graphs  $F_1$ - $F_{20}$ .

## REFERENCES

- [1] R Balakrishnan, The energy of a graph, Linear Algebra and its Applications, 387 (2004) 287-295.
- [2] C. A. Coulson, On the calculation of the energy in unsaturated hydrocarbon molecules, Proc. Cambridge Phil. Soc. 36 (1940) 201-203.
- [3] C. D. Godsil, AZgebraic Combinatorics, Chapman and Hall, New York, 1993.
- [4] I. Gutman, The energy of a graph, Ber. Math. Statist. Sect. Forschungsz. Graz, 103 (1978) 1-22.
- [5] I. Gutman, X. Li, Y. Shi, Graph Energy. New York, Springer, 2012.
- [6] H. W. Kroto, J. R. Heath, S. C.O'Brien, R. F.Curl, R. E. Smalley, C<sub>60</sub>: Buckminster fullerene, Nature 318 (1985) 162-163.
- [7] H. W. Kroto, J. E. Fichier, D. E Cox, The Fullerene, Pergamon Press, New York 1993.

## Appendix

(Energy of small fullerenes)

Fullerene	$E(F)$	Fullerene	$E(F)$	Fullerene	$E(F)$	Fullerene	$E(F)$
C20-1	29.4166	C38-11	58.7204	C40-39	61.5808	C42-44	64.9114
C24-1	36.0214	C38-12	58.5288	C40-40	61.8244	C42-45	64.7128
C26-1	39.7418	C38-13	58.646	C42-01	64.7624	C44-01	68.0308
C28- $d_2$	42.6702	C38-14	58.5704	C42-02	64.999	C44-02	67.8748
C28- $td$	43.106	C38-15	58.7296	C42-03	64.8508	C44-03	68.0584
C30-1	45.904	C38-16	58.4208	C42-04	64.7896	C44-04	68.1098
C30-2	46.086	C38-17	58.432	C42-05	64.7626	C44-05	67.9518
C30-3	45.7036	C40-01	61.6086	C42-06	65.0216	C44-06	67.9898
C32-1	49.0344	C40-02	61.7482	C42-07	64.7776	C44-07	61.9092
C32-2	49.211	C40-03	61.567	C42-08	64.8954	C44-08	68.0356
C32-3	48.9338	C40-04	61.6784	C42-09	64.9758	C44-09	67.9442
C32-4	49.1502	C40-05	61.8184	C42-10	64.7942	C44-10	67.9886
C32-5	49.1904	C40-06	61.7386	C42-11	64.7472	C44-11	68.1398
C32-6	48.8586	C40-07	61.7872	C42-12	64.7472	C44-12	68.1058
C34-1	52.1922	C40-08	61.6436	C42-13	64.9852	C44-13	67.8936
C34-2	52.1984	C40-09	61.626	C42-14	64.8168	C44-14	67.9086
C34-3	52.255	C40-10	61.758	C42-15	60.7944	C44-15	68.0932
C34-4	52.4594	C40-11	61.597	C42-16	64.8014	C44-16	68.0106
C34-5	52.3058	C40-12	61.7432	C42-17	64.9298	C44-17	67.9042
C34-6	52.2574	C40-13	61.803	C42-18	64.896	C44-18	68.0022
C36-01	55.5346	C40-14	61.8172	C42-19	64.8874	C44-19	68.1542
C36-02	55.3592	C40-15	61.7318	C42-20	64.892	C44-20	67.9212
C36-03	55.4138	C40-16	61.6166	C42-21	64.9112	C44-21	68.112
C36-04	55.5568	C40-17	61.6948	C42-22	65.0064	C44-22	67.9726
C36-05	55.3922	C40-18	61.6882	C42-23	64.9694	C44-23	68.0528
C36-06	55.3204	C40-19	61.7938	C42-24	64.9346	C44-24	67.9248
C36-07	55.2206	C40-20	62.0056	C42-25	65.04	C44-25	68.0444
C36-08	55.2962	C40-21	61.8336	C42-26	65.0314	C44-26	68.0846
C36-09	55.489	C40-22	61.9276	C42-27	64.988	C44-27	68.0208
C36-10	55.188	C40-23	61.7366	C42-28	64.8318	C44-28	68.0584
C36-11	55.2028	C40-24	61.8446	C42-29	65.0636	C44-29	67.9508
C36-12	55.4756	C40-25	61.7576	C42-30	64.8208	C44-30	67.9208
C36-13	55.243	C40-26	61.7226	C42-31	64.874	C44-31	68.1298
C36-14	55.1796	C40-27	61.8264	C42-32	65.0274	C44-32	67.956
C36-15	55.2436	C40-28	61.711	C42-33	65.0272	C44-33	67.9274
C38-01	58.4956	C40-29	61.752	C42-34	65.0164	C44-34	68.0778
C38-02	58.2852	C40-30	61.8504	C42-35	64.9186	C44-35	67.9774
C38-03	58.7156	C40-31	61.7256	C42-36	64.9576	C44-36	67.9966
C38-04	58.4652	C40-32	61.808	C42-37	64.805	C44-37	68.1186
C38-05	58.6814	C40-33	61.8934	C42-38	64.9926	C44-38	68.023

C38-06	58.4214	C40-34	61.7598	C42-39	64.8798	C44-39	68.132
C38-07	58.5656	C40-35	61.775	C42-40	65.013	C44-40	68.0286
C38-08	58.5772	C40-36	61.775	C42-41	64.8646	C44-41	68.1408
C38-09	58.4472	C40-37	61.7138	C42-42	64.8646	C44-42	68.175
C38-10	58.6022	C40-38	61.6006	C42-43	64.838	C44-43	68.174
C44-44	67.984	C46-001	71.0836	C46-047	71.252	C46-093	71.2356
C44-45	67.977	C46-002	71.2418	C46-048	71.3736	C46-094	71.414
C44-46	68.1696	C46-003	71.1118	C46-049	71.2156	C46-095	71.3578
C44-47	68.0774	C46-004	71.2406	C46-050	71.2434	C46-096	71.141
C44-48	67.9938	C46-005	71.2382	C46-051	71.1882	C46-097	71.2156
C44-49	68.1404	C46-006	71.1576	C46-052	71.2148	C46-098	71.2516
C44-50	68.034	C46-007	71.0992	C46-053	71.2286	C46-099	71.2578
C44-51	68.1342	C46-008	71.1036	C46-054	71.2546	C46-100	71.2714
C44-52	68.1016	C46-009	71.2118	C46-055	71.2034	C46-101	71.2394
C44-53	68.142	C46-010	71.271	C46-056	71.2076	C46-102	71.1076
C44-54	68.135	C46-011	71.1928	C46-057	71.384	C46-103	71.2234
C44-55	68.0198	C46-012	71.1122	C46-058	71.2704	C46-104	71.274
C44-56	68.0684	C46-013	71.1908	C46-059	71.228	C46-105	71.2722
C44-57	68.0606	C46-014	71.0702	C46-060	71.2462	C46-106	71.2808
C44-58	68.0134	C46-015	71.1748	C46-061	71.2922	C46-107	71.2914
C44-59	68.195	C46-016	71.1164	C46-062	71.2338	C46-108	71.2576
C44-60	68.1242	C46-017	71.189	C46-063	71.239	C46-109	71.1922
C44-61	67.8902	C46-018	71.1682	C46-064	71.2256	C46-110	71.316
C44-62	68.1984	C46-019	71.2542	C46-065	71.321	C46-111	71.1174
C44-63	68.0976	C46-020	71.1288	C46-066	71.1136	C46-112	71.233
C44-64	67.9252	C46-021	71.2656	C46-067	71.2926	C46-113	71.1544
C44-65	68.0242	C46-022	71.1362	C46-068	71.3516	C46-114	71.347
C44-66	67.9318	C46-023	71.2278	C46-069	71.2698	C46-115	71.4664
C44-67	68.223	C46-024	71.2766	C46-070	71.388	C46-116	71.3386
C44-68	68.1042	C46-025	71.1106	C46-071	71.2726	C48-001	74.3358
C44-69	68.0486	C46-026	71.2042	C46-072	71.1162	C48-002	74.1832
C44-70	68.2082	C46-027	71.249	C46-073	71.219	C48-003	74.43
C44-71	68.022	C46-028	71.2722	C46-074	71.2248	C48-004	74.2222
C44-72	67.8262	C46-029	71.2722	C46-075	71.2154	C48-005	74.2454
C44-73	67.9872	C46-030	71.101	C46-076	71.1984	C48-006	74.2548
C44-74	68.1034	C46-031	71.209	C46-077	71.0956	C48-007	74.4236
C44-75	67.9602	C46-032	71.0812	C46-078	71.2272	C48-008	74.3108
C44-76	67.9486	C46-033	71.161	C46-079	71.2462	C48-009	74.268
C44-77	68.1268	C46-034	71.3036	C46-080	71.2442	C48-010	74.2652
C44-78	68.144	C46-035	71.222	C46-081	71.2008	C48-011	74.4414
C44-79	62.0804	C46-036	69.613	C46-082	71.2262	C48-012	74.2786
C44-80	67.9156	C46-037	71.228	C46-083	71.3586	C48-013	74.4128
C44-81	68.0094	C46-038	71.315	C46-084	71.2222	C48-014	74.3184
C44-82	68.2652	C46-039	71.2958	C46-085	71.3056	C48-015	74.3112
C44-83	67.979	C46-040	71.1414	C46-086	71.198	C48-016	74.1876
C44-84	67.976	C46-041	71.14	C46-087	71.2264	C48-017	74.1628

C44-85	68.086	C46-042	71.2362	C46-088	71.2146	C48-018	74.3428
C44-86	68.2144	C46-043	71.0924	C46-089	71.3816	C48-019	74.2846
C44-87	68.1628	C46-044	71.176	C46-090	71.2378	C48-020	74.2298
C44-88	68.204	C46-045	71.2	C46-091	71.304	C48-021	74.2412
C44-89	68.1814	C46-046	71.201	C46-092	71.2694	C48-022	74.2596
C48-023	74.2722	C48-069	74.4688	C48-115	74.252	C48-161	74.385
C48-024	74.3878	C48-070	74.5096	C48-116	74.3756	C48-162	74.3482
C48-025	74.4494	C48-071	74.2846	C48-117	74.2602	C48-163	74.2522
C48-026	74.3512	C48-072	74.3736	C48-118	74.325	C48-164	74.368
C48-027	74.2842	C48-073	74.3116	C48-119	74.365	C48-165	74.434
C48-028	74.2758	C48-074	74.3002	C48-120	74.3082	C48-166	74.3556
C48-029	74.405	C48-075	74.5152	C48-121	74.3446	C48-167	74.4942
C48-030	74.3364	C48-076	74.3102	C48-122	74.34	C48-168	74.2316
C48-031	74.274	C48-077	74.363	C48-123	74.4338	C48-169	74.2938
C48-032	74.3212	C48-078	74.259	C48-124	74.4454	C48-170	74.4346
C48-033	74.3022	C48-079	74.4358	C48-125	74.2872	C48-171	74.3938
C48-034	74.2882	C48-080	74.3932	C48-126	74.3556	C48-172	74.4172
C48-035	74.3144	C48-081	74.2784	C48-127	74.3146	C48-173	74.2542
C48-036	74.4576	C48-082	74.3096	C48-128	74.4594	C48-174	74.517
C48-037	74.2978	C48-083	74.3908	C48-129	74.5126	C48-175	74.2464
C48-038	74.4256	C48-084	74.2696	C48-130	74.3534	C48-176	74.3618
C48-039	74.3418	C48-085	74.298	C48-131	74.477	C48-177	74.3132
C48-040	74.2844	C48-086	74.3296	C48-132	74.5118	C48-178	74.3174
C48-041	74.3846	C48-087	74.3576	C48-133	74.2892	C48-179	74.384
C48-042	74.4232	C48-088	74.3388	C48-134	74.3606	C48-180	74.4598
C48-043	74.3526	C48-089	74.441	C48-135	74.4614	C48-181	74.5078
C48-044	74.3626	C48-090	74.463	C48-136	74.3324	C48-182	74.5156
C48-045	74.2464	C48-091	74.4014	C48-137	74.4538	C48-183	74.5156
C48-046	74.2842	C48-092	74.3696	C48-138	74.3556	C48-184	74.4542
C48-047	74.2858	C48-093	74.4408	C48-139	74.352	C48-185	74.4304
C48-048	74.4108	C48-094	74.4242	C48-140	74.349	C48-186	74.324
C48-049	74.37	C48-095	74.318	C48-141	74.3478	C48-187	74.4964
C48-050	74.35	C48-096	74.4668	C48-142	74.4148	C48-188	74.384
C48-051	74.2682	C48-097	74.2858	C48-143	74.432	C48-189	74.243
C48-052	74.3636	C48-098	74.3756	C48-144	74.3184	C48-190	74.3908
C48-053	74.3418	C48-099	74.3958	C48-145	74.5136	C48-191	74.4948
C48-054	74.468	C48-100	74.3746	C48-146	74.3696	C48-192	74.313
C48-055	74.4616	C48-101	74.1452	C48-147	74.3904	C48-193	74.3614
C48-056	74.2982	C48-102	74.4692	C48-148	74.4976	C48-194	74.5214
C48-057	74.2976	C48-103	74.2594	C48-149	74.3306	C48-195	74.421
C48-058	74.3594	C48-104	74.4408	C48-150	74.4862	C48-196	74.2952
C48-059	74.3938	C48-105	74.4368	C48-151	74.537	C48-197	74.4876
C48-060	74.328	C48-106	74.464	C48-152	74.2266	C48-198	74.2874
C48-061	74.4002	C48-107	74.3382	C48-153	74.3128	C48-199	74.3896
C48-062	74.317	C48-108	74.285	C48-154	74.379	C50-001	77.4112
C48-063	74.4018	C48-109	74.4344	C48-155	74.4148	C50-002	77.3834
C48-064	74.3086	C48-110	74.3566	C48-156	74.3106	C50-003	77.4694



C48-065	74.3294	C48-111	74.279	C48-157	74.2504	C50-004	77.5782
C48-066	74.3974	C48-112	74.2514	C48-158	74.4724	C50-005	77.6014
C48-067	74.3848	C48-113	74.2922	C48-159	74.3824	C50-006	77.4654
C48-068	74.4334	C48-114	74.4856	C48-160	74.3372	C50-007	77.3646
C50-008	77.5168	C50-054	77.6266	C50-100	77.5172	C50-146	77.4804
C50-009	77.399	C50-055	77.5892	C50-101	77.597	C50-147	77.658
C50-010	77.476	C50-056	77.5402	C50-102	77.4358	C50-148	77.6498
C50-011	77.4534	C50-057	77.498	C50-103	77.4842	C50-149	77.626
C50-012	77.4808	C50-058	77.4088	C50-104	77.5366	C50-150	77.6036
C50-013	77.5432	C50-059	77.458	C50-105	77.585	C50-151	77.4758
C50-014	77.445	C50-060	77.5304	C50-106	77.5994	C50-152	77.6116
C50-015	77.5954	C50-061	77.4794	C50-107	77.5508	C50-153	77.5106
C50-016	77.4068	C50-062	77.5634	C50-108	77.5324	C50-154	77.5458
C50-017	77.5372	C50-063	77.4488	C50-109	77.5904	C50-155	77.6512
C50-018	77.4138	C50-064	77.4348	C50-110	77.4896	C50-156	77.383
C50-019	77.4294	C50-065	77.546	C50-111	77.4794	C50-157	77.5212
C50-020	77.5092	C50-066	77.5558	C50-112	77.6324	C50-158	77.5002
C50-021	77.3932	C50-067	77.434	C50-113	77.4222	C50-159	77.6406
C50-022	77.3984	C50-068	77.6372	C50-114	77.4964	C50-160	77.5108
C50-023	77.3684	C50-069	77.521	C50-115	77.5056	C50-161	77.5392
C50-024	77.4282	C50-070	77.4988	C50-116	77.3836	C50-162	77.3912
C50-025	77.5822	C50-071	77.4498	C50-117	77.6148	C50-163	77.4862
C50-026	77.5738	C50-072	77.5956	C50-118	77.6284	C50-164	77.4942
C50-027	77.5956	C50-073	77.6224	C50-119	77.528	C50-165	77.5086
C50-028	77.514	C50-074	77.5076	C50-120	77.5342	C50-166	77.526
C50-029	77.4316	C50-075	77.5714	C50-121	77.6052	C50-167	77.3888
C50-030	77.4222	C50-076	77.5838	C50-122	77.5856	C50-168	77.3894
C50-031	77.4564	C50-077	77.5288	C50-123	77.5818	C50-169	77.6386
C50-032	77.4654	C50-078	77.4692	C50-124	77.6336	C50-170	77.5914
C50-033	77.511	C50-079	77.6306	C50-125	77.6232	C50-171	77.4724
C50-034	77.4656	C50-080	77.6342	C50-126	77.5926	C50-172	77.536
C50-035	77.5196	C50-081	77.5614	C50-127	77.4748	C50-173	77.5542
C50-036	77.532	C50-082	77.4404	C50-128	77.6558	C50-174	77.6276
C50-037	77.4812	C50-083	77.4536	C50-129	77.4352	C50-175	77.6192
C50-038	77.4334	C50-084	77.612	C50-130	77.4564	C50-176	77.4412
C50-039	77.516	C50-085	77.4434	C50-131	77.47	C50-177	77.5892
C50-040	77.609	C50-086	77.442	C50-132	77.4602	C50-178	77.5698
C50-041	77.4814	C50-087	77.4338	C50-133	77.5542	C50-179	77.5694
C50-042	77.4434	C50-088	77.4262	C50-134	77.5408	C50-180	77.583
C50-043	77.4184	C50-089	77.6254	C50-135	77.5616	C50-181	77.5104
C50-044	77.6338	C50-090	77.6628	C50-136	77.624	C50-182	77.5582
C50-045	77.4584	C50-091	77.5612	C50-137	77.559	C50-183	77.6498
C50-046	77.4444	C50-092	77.4726	C50-138	77.4552	C50-184	77.6274
C50-047	77.5484	C50-093	77.604	C50-139	77.4598	C50-185	77.5564
C50-048	77.486	C50-094	77.5254	C50-140	77.643	C50-186	77.4526
C50-049	77.509	C50-095	77.4038	C50-141	77.4794	C50-187	77.6424

C50-050	77.452	C50-096	77.4232	C50-142	77.5274	C50-188	77.4992
C50-051	77.5422	C50-097	77.5648	C50-143	77.4758	C50-189	77.5746
C50-052	77.3836	C50-098	77.4804	C50-144	77.4122	C50-190	77.4216
C50-053	77.5838	C50-099	77.3978	C50-145	77.5246	C50-191	77.6152
C50-192	77.5694	C50-238	77.4314	C50-193	77.4488	C50-239	77.6504
C50-194	77.4878	C50-240	77.5294	C50-195	77.4616	C50-241	77.5374
C50-196	77.688	C50-242	77.6678	C50-197	77.547	C50-243	77.6686
C50-198	77.5782	C50-244	77.5672	C50-199	77.6014	C50-245	77.6826
C50-200	77.534	C50-246	77.5346	C50-201	77.5678	C50-247	77.5516
C50-202	77.4596	C50-248	77.584	C50-203	77.5368	C50-249	77.6394
C50-204	77.5238	C50-250	77.58	C50-205	77.4606	C50-251	77.6492
C50-206	77.4818	C50-252	77.5456	C50-207	77.4436	C50-253	77.6004
C50-208	77.3798	C50-254	77.4972	C50-209	77.5044	C50-255	77.4572
C50-210	77.4854	C50-256	77.4404	C50-211	77.4392	C50-257	77.5366
C50-212	77.601	C50-258	77.7308	C50-213	77.5784	C50-259	77.5816
C50-214	77.4084	C50-260	77.5606	C50-215	77.4132	C50-261	77.384
C50-216	77.508	C50-262	77.5464	C50-217	77.4792	C50-263	77.5404
C50-218	77.467	C50-264	77.6166	C50-219	77.3992	C50-265	77.6356
C50-220	77.4486	C50-266	77.5108	C50-221	77.5378	C50-267	77.5576
C50-222	77.6178	C50-268	77.4036	C50-223	77.4692	C50-269	77.4862
C50-224	77.508	C50-270	77.37	C50-225	77.4422	C50-271	77.5788
C50-226	77.3706	C50-227	77.6582	C50-228	77.5572	C50-229	77.639
C50-230	77.562	C50-231	77.5438	C50-232	77.4622	C50-233	77.5782
C50-234	77.418	C50-235	77.5112	C50-236	77.4776	C50-237	77.6194